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An Objective in Education*

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A Philosophy

Early in February a mimeographed letter was sent to more than a hundred national and sectional officers of the Mathematical Association of America asking for suggestions in formulating the policy of the Association with respect to post-war educational problems. The first and primary question was, "Do you believe that the Association should, through the action of its committees, keep in close touch with developments in the educational world and take an active part in moulding opinion?" The reply to this question was an emphatic and unanimous "yes." Since those replying undoubtedly constitute a fair sample of the total membership of the Association, the officers feel that they have a mandate for action.

In these days when education seems to have so many aims and objectives, it was interesting to note that the members of the Association are pretty thoroughly agreed upon a fundamental philosophy of education. This is not as one might perhaps have believed that all students be required to take mathematics. It is, on the contrary, the belief that education should be tailored to the capacity of the individual. In this belief the members of the Association seem to have the support of many educators in other fields.

While this talk derived much inspiration from the replies to the above mentioned letter, I wish to assume full responsibility for the remarks that follow. The Association is in no degree committed to agree with them.

A Problem

The philosophy of an education fitted to the capacity of the student has, up to the present, been largely developed in the interests of the weaker students. This is of course an essential step in the unique experiment in universal education for which the schools of the United States are now a vast laboratory. It is an experiment which would be attempted only by a great democracy, and it is an experiment which must be successfully completed if democracies are to survive and flourish.

But in solving the problem of the poorer student, difficulties have been placed in the path of the superior student which in some instances make it difficult for him to obtain the education of which he is capable. Because the superior students are in the minority, and because they do not become problem children, they are often

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woefully neglected. Because advanced subjects in a small high school have small registration, they are often abandoned and the time of the teacher is "more economically employed" with large classes of "future citizens."

It seems evident that the people of the United States do not see with sufficient clarity that the education of leaders is an objective compared with which the education of the masses is of lesser importance. This may sound like heresy in modern America, but it is an historical fact. Nations have become prominent and prosperous without universal education, but no nation has ever been able to maintain its position as a civilized community without a group of talented and educated leaders. America can and must solve both problems.

Let us recall that the history of the world is largely the history of individual men, many of whom are remembered only through their works. Modern civilization has been developed and is being maintained by relatively few persons. For each branch of science, the arts, philosophy, government, and military science it would be possible to name a hundred men without whose contributions the world would be appreciably poorer. Where would American technology now be if every person listed in the biographical book *American Men of Science* had failed to attend college? It is interesting to speculate on the present state of our civilization if one man in a thousand, since 10,000 B.C., properly selected, had failed to mature intellectually. It is quite possible that we should still be roaming the woods in skin clothing. Whether we would be happier in that condition than we now are is beside the point.

It is pertinent to note that the present apex of American military might was achieved without compulsory peace-time military training. A vitally important factor in our success was the work of American scientists in developing new weapons of offense and defense. Indeed, this was probably the decisive factor in the war—a fact which will some day be clearer than it is now to the general public. If compulsory military training has the effect of delaying or in any way interfering with the development of our skilled scientists, the act will be as disastrous as the destruction of the proverbial goose with the talent for laying golden eggs.

Even if we suppose that the gifted student eventually reaches the university, the years of time which he has lost are never recovered. In no direction is American wastefulness more costly than in the waste of precious years in the schooling of the superior child. Contrary to popular superstition, it does not injure the brain of a child to allow him to absorb knowledge at his natural rate of speed, even though this be five times the rate of his slightly retarded classmates. On the other hand, to allow a superior child to coast along without the exercise of effort is distinctly detrimental.

The fact that Europe has produced men of great genius more abundantly than has America cannot be due to differences in native ability, for we are of the same stock. It is clearly due to the fact that the better European schools are geared to the pace of students of higher average ability, and about two years are saved in their elementary education. These two precious years which American students lose are never recovered, and are subtracted from their years of greatest productivity.

There are some who, with a befuddled conception of the meaning of democracy, argue that a special education for the gifted means the production of an intellectual aristocracy and is therefore undemocratic. The effect of such thinking is to limit all opportunities of leadership to those men whose families are sufficiently wealthy to send their sons to private schools and colleges, a plan which is wasteful of precious talent and is the very antithesis of democracy. Genius does frequently appear among the children of the poor, and a truly democratic educational

system will provide adequate education for the development of such genius.

In advocating the double-track curriculum in the secondary schools, we must be careful not to call the express track the "college preparatory" track. A superior student in high school has a right to a superior education even if he is financially unable to attend college. The fact that his financial situation is more subject to alteration than is his mental quipment should impel a teacher to be very cautious about advising a capable student to elect a vocational course. Many of life's little tragedies occur when students discover that their high school work is of no use to them in preparation for their college work.

A Solution

It is far easier to state a problem than to solve it, and I have to approach this problem cautiously. The City of New York has reached a satisfactory solution with its highly trained and adequately paid teachers, its special schools for the gifted as well as for the handicapped, and its system of city colleges which admit only those graduates of the local high schools who have attained high grades in a substantial college-preparatory course. But the small high schools of northern Wisconsin, for instance, present a problem which cannot be solved in the same manner.

One possibly satisfactory answer to this problem would be in the establishment of a few centralized schools throughout the state which would take the better student at the tenth grade level and fit him for college. This idea is not new, it is, to take one instance, the motivation of the establishment of the college at the University of Chicago. President Hutchins sees very clearly that the last two years of high school are frequently very wasteful of a talented student's precious years of youth, and he has provided an answer.

A means of accomplishing this end might be the junior college offering four years of work from the eleventh to the fourteenth grades inclusive. The junior college movement had considerable impetus a few years ago and this has not been entirely lost. But if the junior college is merely to offer a diluted form of college education for the student who is too weak for the university, it will not serve the purpose with which we are now concerned. Such an institution would merely extend the student's high school education by two years. If the junior college is to be the solution of the problem of the superior student, it must have high entrance requirements, a substantial curriculum from which the unessential courses have been eliminated, and a competent faculty who are able to stimulate and enthuse the students.

At the beginning of the eleventh grade the tone of the school should change from the juvenile to the adult. The student by this time should be made to realize that scholarship is a dignified career, fully as worthy of masculine attention as football or class politics. That the scholars are the people who keep the sacred fire of culture burning is a fact which merits as much time for indoctrination as the nutritional value of cheese.* The age in which we live is a serious age. Competition between men and nations is deadly. The transition which our eighteen-year-olds had to make from the childish atmosphere of the school to the brutality of army life might have been easier if the schools had not for years pretended that realities need not be faced, that difficult subjects may be skipped, that students who fail must nevertheless be passed along for fear of consequences to the personality. The epoch of such nonsense is past, and I believe that the American people are now in a mood to face facts, and to demand that their children be prepared in the schools to face life as it is, not a fairy-land life in which one can avoid the consequences of stupidity and laziness.

* There is an unenforced law in Wisconsin that all pupils shall be instructed each week in the salubrity of dairy products.

A Liberal Education

For many years a college education meant the traditional course in liberal arts with Latin and Greek and the Roman and Hellenic cultures as a background. It was and still is a most excellent course, demanding that the student give his best efforts and rewarding him with a stereoscopic view of our language, history and civilization. The universities must preserve this course for those who will take it.

In the last two hundred years, our civilization has completely changed. Sciences which were unknown in 1700 now determine the fate of nations and guide the daily lives of men. While the traditional liberal arts course is still important, it is not sufficient as it once was to interpret to the individual the world in which he lives. If the universities would reexamine and restate their objectives, perhaps the high schools would be able to see more clearly what their procedure should be.

I believe that from now on the central course of study in the colleges should be a course in liberal science not leading to any specialization but serving to interpret to the student the marvelous civilization of which we all are conscious, but which is incomprehensible at least in part to most of us. I can see the raising of eyebrows by my colleagues in the humanities, the familiar charge that the mere scientist is a technician, not a truly educated man who feels the finer things of life. I am not impressed. Unless the humanitarian understands the forces which keep the earth in its orbit, he is not a truly educated man. The pure sciences, taught with the objective of explaining basic principles, have as much aesthetic value as the humanities, and they have a grandeur all their own.

This course in liberal science should include work in physics and chemistry, and at least an introduction to astronomy, geology, biology and physiology. These should not be of the I-can't-tell-you-why-because-you-wouldn't-understand-it type of course, but courses where facts are proved. Astronomy, that now neglected gem of the sciences, should be highly recommended. There is no course which the humanitarians can offer which has quite the same power to orient a student with respect to his environment and reduce him to a position of humility as astronomy. Mathematics through differential equations is, of course, essential for all the physical sciences.

It goes without saying that the course should include English and a modern language and history, but possibly all of these courses can be given the emphasis demanded by their position in a course in liberal science. Composition and speech remain important, but the detailed study of English literature may have to be sacrificed as was the literature of classic times. Similarly language study should give more attention to the scientific literature than to archaic forms. History should include a description of modern research in pre-history and much emphasis on the history of science. In fact, a course in the senior year in the history of science given to students who are familiar with the sciences should be one of the high spots which make the course a success. Have you a pet course which should be included? There is room for electives.

An engineering course of two years' duration following a course in liberal science such as I have described would produce a man well-rounded and generally educated, and technologically superior. The investment of an extra two years of study would be well worth while and as a matter of fact the double-track scheme of elementary and high school education could save for the superior child the two extra years required.

The vogue of education-by-stuffing is definitely on the wane. The great corporations who now take so many of the graduates of our engineering schools have come to desire the students of superior ability who have done well in the basic science courses. Courses in engineering practice are of some importance, but many corporations prefer to initiate the junior engi-

neer into their own methods and techniques. Mathematics and the basic sciences are the subjects which count. All this argues in favor of the course in liberal science.

Mathematics

I am not going to put forth any arguments in favor of the teaching of mathematics in our public schools. If there is anyone present who does not realize that we are living in the age of science, and that science rests on a mathematical foundation, I know of no language by which I can reach him. There are in our schools students who cannot learn mathematics. It is doubtful if they are capable of fundamental thinking in any direction. They are destined to be drawers of water and hewers of wood in our civilization. They should be given generous and sympathetic attention so that they may become useful citizens. They will not become leaders.

There are other students who have in some way acquired a distaste for mathematics and a conviction that they are unable to master it. They are the casualties of poor teaching. Under normal conditions they pass through life firm in the conviction that they are not mathematically minded. During the days of the ASTP, I encountered numerous instances which showed exactly where the trouble lay. These boys have many times assured me that they hated mathematics, had no ability for it, and that only the power of the Army of the United States could persuade them to take it. But in spite of the fact that the ASTP curriculum was an educational monstrosity, many of these boys became mathematical fans. Recently I received two letters from the Pacific theatre from boys who had been in my ASTP class in calculus. One stated that he wanted to return to Wisconsin and take all the courses in mathematics and chemistry that were given. The other asked for the title of a book on non-Euclidean geometry.

If I were asked for suggestions for the improvement of the teaching of mathematics in high school, I should say that we need more teachers who have a burning enthusiasm for the subject. If they love the subject for its own sake, they will continue to study it during their teaching years, and they will be able to communicate their enthusiasm to the student. Certainly mathematics is important and useful, but more than that, it is fun. This is the motive which, more often than any other, determines a person's choice of vocation. Many majors in the university are determined by the personality of a favorite teacher.

Those of us who appreciate mathematics should insist that more mathematics, not less, be offered in the post-war schools and that all students who can profit by it be encouraged to partake. There is strong evidence that the war has awakened America from its world of fancy and that the country is ready to face realities. Mathematics has been belittled by some educational experimenters in the last twenty years, but the demands of the war have brought confusion upon them. As Kipling stated it,

> It's 'Tommy' this, and 'Tommy' that,
> And 'Throw him out, the brute!'
> But it's 'Savior of his country'
> When the guns begin to shoot.

Looking Ahead

The four years of mathematics, which high schools that wish to be designated as college-preparatory must offer, should prepare a student to begin calculus upon entering college. In no other way can the student who takes physics in his freshman year avoid the loss of a year. Physics taught without calculus is a lean and ineffectual thing, and I sometimes believe that it does more harm than good in fixing rationalizations rather than fundamental methods in the student's mind. The synchronization of courses in physics and mathematics is one of the problems now crying for solution. This was done to some extent in the Army and Navy programs

and was, I believe, one of the high spots in the programs.

Analytics is now being taught in the fourth year in some of our large city high schools, in many preparatory schools, and in the schools of Ontario. It is not an untried experiment. It supplants the traditional course in advanced algebra, accomplishes the same objective as regards manipulative skill, and by furnishing a motive, increases the interest of the student.

There is one objection which is brought up against the teaching of analytic geometry in high school, and that is, as one man expressed it, "The first persons to study it would have to be the teachers." If that is true, I think I am in favor of putting analytic geometry into all the high schools, for I am sure that all high school teachers of mathematics should know analytics. If teachers were recruited from the ranks of those who had taken the course in liberal science which I described, they would be thoroughly qualified in technical skill, breadth of vision and, I believe, enthusiasm.

Summary

It appears that more and more educators are abandoning the thesis that all students should be forced to go as far as they can through the college preparatory high school courses. The principle of the course tailored to fit the capacity of the student seems to be attracting more and more adherents, not only among high school teachers, but also among college teachers. This plan of a double-track curriculum promises excellent results provided the express track is not neglected in favor of the freight track, and provided that students of ability are persuaded to purchase Pullman tickets and not bills of lading.

There can be no real permanent threat to the teaching of mathematics in our schools, since our culture is essentially a scientific culture resting upon a mathematical base. The best way that mathematics teachers can circumvent ill considered attacks upon mathematics is to support sound educational principles in general without too much emphasis upon mathematics in particular. If collegiate instruction were built around a non-specialized course designed to interpret modern civilization to the student, such a course would necessarily contain courses in science and mathematics. It would require in preparation a full four years of mathematics in high school, and would provide an ideal training for prospective high school teachers of science and mathematics.

Irrespective of how our problems are eventually to be solved, it seems desirable that teachers of mathematics at all levels should now work together in the interests of sound education, and should join forces with persons of similar ideals and objectives in other branches of learning. United behind well-defined objectives, the educational societies of America will be able to keep education in this country on a rational basis.

Notice to Members of the National Council!

The annual meeting of the National Council of Teachers of Mathematics will be held in Cleveland, Ohio on February 22 and 23, 1945. President Wren has not yet announced hotel headquarters. Look in the January and February (1946) issues for further details—Editor.

Geometry and Mr. Newell*

By L. E. C.

I. *E.T.H.S. and Mr. Newell's class room.* I started to study geometry in the first decade of the twentieth century. I was a junior in Evanston Township High School and Mr. Newell was my teacher. Now, that I am to some extent in his position then, a teacher of high school geometry, I should like to set down some of the values that have come to me through his class, and how they came.

Evanston Township High School was founded in 1883 and Mr. Boltwood was the principal. He was a small white-haired old gentleman whose strong personality was at first concealed by his frail appearance. Rumor had it that he knew as much as all our departmental teachers put together, for couldn't he go into any class and teach that class better than the teacher? I remember well his customary reply to those called on to recite, those unlucky ones who responded with the old student excuse.

"I know it but I can't say it."

"If you can't say it you don't know it," he would snap back. What he would have thought of professional judgments based entirely on "true-false" or "multiple-choice" questions, I hesitate to imagine!

At this time there were, as I remember it, two mathematics teachers in our high school, Miss Barr to teach freshman algebra and young Mr. Newell to teach sophomore algebra and junior geometry. I remember that I looked forward to studying geometry. Algebra had been a joy to me in that it freed me from the petty accuracies of arithmetic. Geometry would probably be even nicer. This expectation was amply justified because it was a branch of mathematics and because of our teacher.

As I recall the class room, I remember two things, proofs of propositions and solution of problems. On many days the boards were filled with the proofs of propositions, those which had just been assigned and those which we were reviewing. But the big thing in that class was the problems which we solved, they were the mentally exciting part of geometry. Mr. Newell didn't give us one or two each day out of the text book. He handed out mimeographed sets of "originals" three or four times during the year, "handed them out as if he were giving us something precious, as indeed he was" says one of my fellow classmates. These problems were due to be solved not tomorrow but at a not too distant future date. We were keenly interested to see each set as it came to us, to decide which would be the difficult problems and to hear our companions' reactions to them.

It soon became evident that there were two kinds of originals, those which were set up with the phrase "to prove" and those which read "to construct." At that time I liked the proof problems better, but even then I sensed that Mr. Newell liked the construction problems better.

One of my classmates, L. R., now herself a high school mathematics teacher has said:

"Somehow or other Mr. Newell got across to us that the chief function of a mathematics student in high school was to *solve problems;* that if we solved mathematics problems in high school we would thereby be prepared to solve adult problems in *any* field."

She went on to say that there was no other class where she felt as if her personal-

*** It is only fitting that this article concerning Mr. Newell should appear in this jubilee number as he was one of those who was interested in helping to found the National Council of Teachers of Mathematics.—Editor.**

ity (as expressed in her mind) was so "glorified" and respected as in his class. She continued:

"He used to have one afternoon a week for conference and I found my greatest joy in working my exercises there with other students. I really found my social life in doing my lessons with my classmates because Mr. Newell did not object to having his pupils talk over things with him and with each other as they worked."

R. B., another classmate, now a high school physics teacher, tells me that she enjoyed studying with Mr. Newell because, more than any other teacher she knew, he contrived to get across to his pupils that their education depended on their own activity and he was merely there to help. She says she has a vivid recollection of his assigning certain problems for a lesson in trigonometry, saying "If you need any help call on me," and putting his feet on the desk to read his paper!

So much for our childhood recollections of this teacher and his classes. But before these theorems could be assigned and these mimeographed problems attacked, before the trigonometry teacher could put his feet on the desk and say "If you need any help come to me," before all this could take place, every intelligent adult suspects, and every good teacher knows that much in the way of general preparation must have gone on. In trying to reconstruct this part of the classwork for my own help and others, I found I had absolutely no specific recollection of these general preparations except a lesson one day as to the dual nature of a line—either a line segment which could be imagined as extending out into space north and south or east and west, or a line segment which was really an arc of a great circle girdling the earth. Fortunately in recalling the general preparations of other topics, I have been able to refer to Mr. Newell's text books and shall give some detailed study of them in the next section.

II. *Text-Book Studies.* The sets of problems given out by Mr. Newell have been mentioned. From compiling these mimeographed pages he passed easily to a 50 page pamphlet of well-chosen geometry originals. This was published about 1908. His first book written in cooperation with George A. Harper of New Trier High School was published in 1915. Two other editions, bearing the name of Mirick, Newell and Harper, appeared in 1929 and 1935. It is from these text books that I am able to reconstruct Mr. Newell's general preparation of his classes for their problems and theorems.

In these books his appreciation of the value of the history of mathematics is immediately evident. However, to any geometry teacher, there are three challenges which I have always felt that he met exceptionally well both by his class room teaching and by the printed word, three difficult topics in Plane Geometry whose introduction I wish to review from these books. The topics are:

1. The concept of locus
2. The solution of construction problems with the aid of a preliminary sketch of the desired construction
3. The solution of proof problems by analysis.

To begin to prepare the way for the concept of locus Mr. Newell writes:

"Draw two parallel lines and find two points each equally distant from them.

"Draw the line of these two points. How is every point on this line related to the parallels? Is any point, not on this line, equidistant from the parallels? etc."

Then *follows* the definition of locus.

This occurs just after the proposition concerning any point on the bisector of an angle, i.e. in book I, the book on straight lines. The new word "locus" is used immediately not merely as a word which will help us to state a complicated theorem more easily but as a tool to assist us in solving construction problems. This tool is further developed informally later in the straight line book and then picked up and

used easily in the circle book.

As to solving construction problems he says:

"If a method of construction is not readily determined, draw a free hand figure on scratch paper fulfilling the conditions of the problem, and attempt by analysis to discover the necessary relations."

To me this sentence should be italicized for every high school geometry pupil. If our problem is solvable, if our edifice can be built under the given conditions, if our machine can be invented as we hope, it is only by studying carefully the relation of the parts of the desired result that we can confidently expect to discover *how* to solve the problem, build the edifice, make the machine. All else is haphazard accident.

The third especially valuable development of Mr. Newell's book comes, as I have said, in the development of the power to solve "proof problems" by the technique of analysis.

The start of this technique comes also in Book I beginning with several general directions. Then comes an *illustrative exercise* that emphasizes the technique of assuming the desired truth and finding possible facts which could lead up to it. If this thing, A, is to be proved true, it might be implied in B. If B is to be proved true it might be implied in C. But C is true and so A and B are also true. Don't assume that Mr. Newell in talking to or writing for the high school pupil abstracted the general form of this technique. In his textbook he *illustrates* it and calls attention to its peculiar characteristics. Having preceded his illustrative exercise with the exhortation

"Study carefully the following method of discovering the proof of an exercise" he provides the pupil with a second exercise, *not too difficult*, which may be solved by a similar analysis.

Thus by seemingly casual preparations in Book I and the early part of Book II he has prepared his classes with a method of attack for construction problems and proof problems. All that is left for them to master is the new material of the remaining books. But for problem solving they have a technique that may be applied to geometry problems for the rest of the year and later transferred to other fields. For truly

"Man is born to . . . problems, as the sparks fly upward."

Since the appearance of Eric Temple Bell's *Men of Mathematics*, mathematicians have shown less fear of the idea of being counted as "real persons" so my next question will not (now) seem amiss even in a professional article:

What sort of a person was this Mr. Newell, our geometry teacher and guide into the world of mathematics?

III. *Mr. Newell as a Person.* Marquis J. Newell was born January 11, 1873 and grew up on a farm near Kalamazoo, Michigan. His first teaching was done at the age of 17 in a one room school where there were 30 different recitations to be heard in one day. He graduated from the State Teachers' College in Kalamazoo June 17, 1896 with an A.B. degree. By dint of studying and teaching he gained his master of arts degree from the University of Michigan in June 1900 and came to Evanston Township High School in the fall of that same year. The following summer he returned to Michigan to take a course in teaching geometry with W. W. Beman. It was in this course that he gained his appreciation of problem solving as a major goal in high school geometry. He also returned to his class room with a lasting interest in the history of mathematics.

At this time Evanston was known as the "Athens of the west," the home of many of the finest business and professional people of Chicago. Among many of them there prevailed at that time the tradition that their sons and daughters must go east to college. Gradually this had come to mean going east for preparatory school as well. Mr. Boltwood, the principal of

E.T.H.S. and an eastern man himself, felt this to be a real problem to his high school. Soon after Mr. Newell's arrival in Evanston Mr. Boltwood put the problem before his young mathematics teacher.

"We want to break down this loss of our prospective pupils," he said. "The great stumbling block is mathematics."

Finally Mr. Newell, in his second or third year in Evanston, got hold of a daughter of the X family, a girl headed for Bryn Mawr. He promised he'd get her ready for her college board examinations at her own local high school. He worked with her, practically coached her all through the year, and she passed her entrance examinations with flying colors.

From that time on pupils who had done well in our school had no trouble with eastern examinations, even the dreaded mathematics, and Mr. Newell's boys were welcome even in M.I.T.

Meanwhile Mr. Newell took graduate work in Physics under Compton at the University of Chicago and was active in founding and developing the Men's Mathematics Club of Chicago and vicinity and through that the National Council of Teachers of Mathematics.

As the twentieth century hit its stride, Mr. Newell taught third and fourth year mathematics, then fourth year only, the senior elective mathematics, Trigonometry and College Algebra. These classes were usually composed of 25 or 30 boys who not only thought they were going to college, they knew they were. With them he had the faculty of getting a lot of work done, without irritating either them or their parents. Because they were a picked group he could carry out the old Evanston policy, no grades on effort, all grades on achievement.

Meanwhile Mr. and Mrs. Newell and their family made themselves one with the city of Evanston, joining one of the churches and becoming active in civic affairs as well as developing their own family life.

Speaking of him in his dual role as father and teacher his daughter said:

"He always wanted us to think for ourselves, and, what was implicit in that, he sensed a need for people to work out their own philosophy. If someone did it for you, you rather resented it."

Mr. Newell's genius for friendship and comradeship was evident the moment you stepped into his class room for he was never too busy to greet the returned pupil, or to confer with a colleague. One of these colleagues, speaking of him, said "He was a most generous man, generous and free, with what he had and could do for you."

In 1914 Marquis J. Newell of Evanston High School and Charles M. Austin of Oak Park got their heads together and decided that there might be enough "men of mathematics" in the Chicago neighborhood to form a small club. They planned a very informal group that should eat together about once a month in some loop cafeteria and be addressed by a speaker in their chosen field. From this apparently casual beginning came the "Men's Mathematics Club of Chicago." It had on its roll call such well known names as H. E. Slaught, E. R. Breslich, W. D. Reeve and E. W. Schreiber as well as many others. It was a strong group, this "Men's Mathematics Club of Chicago" and out of their friendship developed the National Council of Teachers of Mathematics.

As the years went on and the mathematics department of E.T.H.S. grew with the school the number of Mr. Newell's fellow teachers increased and so the number of his close comrades and potential friends grew steadily. That these potential friends became real friends is amply attested by the fact that now in his class room is an illuminated manuscript bearing the tribute of these colleagues.

"Marquis Joseph Newell taught mathematics in Evanston Township High School for forty-one years. During that period every student whose primary interest involved mathematics came under his influence. The superior records made by these

students in colleges and in universities did much to establish the high academic reputation which this school enjoys.

Mr. Newell was admirably equipped for his work by natural endowment, wide interests and thorough training. He inspired and set high standards for his associates in teaching as well as for his students. He was an author of textbooks in mathematics and an enthusiastic supporter of organizations devoted to the study and teaching of mathematics and science. Few men knew Evanston as he did. His service as an interpreter of the school to the community was invaluable.

In his teaching philosophy the four cardinal virtues were initiative, accuracy, speed and the habit of work."

At the same time that I studied Geometry under Mr. Newell I studied American Literature under Miss Effyan R. Wambaugh. One of the high spots of the year was the reading of Emerson's "American Scholar."

"The American Scholar is to be the man thinking in contrast to the thinker as he has been in Europe" wrote Emerson, or words to that effect. The subtle essence of personality in our geometry teacher constrains me to write forty years later—here was no European emphasis on intellectual values *per se* or on the intellectual value of mathematics to the exclusion of its other values, but on the humanity of mathematics, its good to the individual and to the race—mathematics which helps us to solve our "felt problems."

Mr. Newell was essentially "a man thinking and not a thinker" and he taught us geometry to make us "women and men thinking and not thinkers."

Here then are the educational values that come to me through "geometry and Mr. Newell" as nearly as I can remember and reconstruct them.

Many people have helped me to write this story; among them are: Mr. Francis Leonard Bacon, Superintendent-Principal of Evanston Township High School, Miss Clara D. Murphy, Chairman of the Department of Mathematics, Mr. H. Dayton Merrell, and Mr. Francis W. Runge, Mr. Newell's colleagues.

Mr. C. W. Austin directed me to the report of the Twentieth Anniversary meeting of the Men's Club and Mr. Newell's daughter, Pauline Newell Christensen, has helped me in many ways. The quotations from the books are made with the permission of Row, Peterson and Co., publishers

National Council Yearbooks!

The National Council Yearbooks are rapidly going out of print. The October issue contained a list of those still available and the prices for which they may be obtained. Teachers who wish complete files and particularly school libraries who lack certain books should act now.—Editor.

Henry Holt & Company, Publishers, 257 Fourth Avenue, New York, N. Y., are printing copies about 14×20 inches on firm, but not stiff paper, of "The River Mathematics," which appeared as the frontispiece in the October (1945) issue of The Mathematics Teacher.

Anyone wishing copies of this picture may secure the same by sending 10¢ to cover the mailing tubes and postage cost, to the above address.

—Editor

The Number e—The Base of Natural Logarithms

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Among the outstanding constants in mathematics e ranks first, even before π, in regard to both the variety of its mathematical implications and the amount of its practical applications. The most outstanding qualities of e show up in the calculus. e is the basis of the exponential function which is unchanged through differentiation. For $y=e^x$ we get $y'=e^x$; $y''=e^x$; $\cdots y^n=e^x$. All derivatives of $y=e^x$ are equal to the original function. The general solution of the differential equation $\frac{dy}{dx}=y$ is $y=C\cdot e^x$ (separating the variables we get $\frac{dy}{y}=dx$; ln $y=x+c$, $y=e^{x+c}$ and for $e^c\equiv C$: $y=C\cdot e^x$). Therefore, all possible functions which are equal to their first derivative and therefore also to their higher derivatives are products of the exponential function with the basis e and a constant factor.

An immediate consequence of this fact regarding e is the expression of e obtained through expansion of the function $y=e^x$ according to MacLaurin's series:

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\cdots\frac{f^{(n)}(0)}{n!}x^n+\cdots.$$

As in the case of $f(x)=e^x$ the derivatives are $f'(x)=f''(x)=f^{(n)}(x)=e^x$ and we obtain for $f(0)=e^0=1$; for $f'(0)=f''(0)=f^{(n)}(0)=1$. For e^x we get the power series

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\frac{x^n}{n!}+\cdots$$

substituting for x the value 1 we have

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots\frac{1}{n!}\cdots$$

which being a convergent series leads to the definite value of $e=2.718281828459\cdots$.

A practical application of the described quality of e is Euler's method to solve linear differential equations, both of the second and of higher orders, through the substitution $y=e^{rx}$. As the linear differential equations of the second order include the outstanding differential equation for harmonic vibrations: $\frac{d^2y}{dx^2}=-a^2y$ there is an almost unlimited field of applications in mechanics, in acoustics, electricity, etc. With the substitution $y=e^{rx}$ we obtain $\frac{d^2y}{dx^2}=r^2e^{rx}$ and for the equation $\frac{d^2y}{dx^2}=-a^2y$: $r^2e^{rx}=-a^2e^{rx}$. This furnishes the auxiliary equation $r^2=-a^2$ the roots of which are $r=\pm ai$. From the roots we get the general solution $y=C_1e^{ai}+C_2e^{-ai}$ to which the form can be given: $y=C_1\cos ax+C_2\sin ax$ by applying the relation $e^{ix}=\cos x+i\sin x$ which in itself represents an outstanding link between e and the trigonometric functions.

From $e^{ix}=\cos x+i\sin x$ and $e^{-ix}=\cos x-i\sin x$ we obtain, through addition $\cos x=\frac{e^{ix}+e^{-ix}}{2}$ and through subtraction $\sin x=\frac{e^{ix}-e^{-ix}}{2}$. In analogy to these functions of x the "hyperbolic" functions were defined as $\cosh x=\frac{e^x+e^{-x}}{2}$ and $\sinh x=\frac{e^x-e^{-x}}{2}$ which in turn led to the Gudermannian function $gdx\equiv\arctan(\sinh x)$ with its application to the Mercator chart of the earth etc.; all are based upon the constant e.

Since our oldest records in the history of mathematics, special attention had been

paid to the sequence: 1; 2; 4; 8; 16; 32; 64; 128; $\cdots$ obtained through continued "duplication" which, up to the sixteenth century, had been considered as a special operation besides multiplication. The differences between the consecutive terms of the sequence 1; 2; 4; 8; $\cdots$ and their differences reproduce the same sequence:

$$\begin{array}{llllllll}
1 & & & & & & & \\
 & >1 & & & & & & \\
2 & & >1 & & & & & \\
 & >2 & & >1 & & & & \\
4 & & >2 & & >1 & & & \\
 & >4 & & >2 & & >1 & & \\
8 & & >4 & & >2 & & >1 & \\
 & >8 & & >4 & & >2 & & \\
16 & & >8 & & >4 & & & \\
 & >16 & & >8 & & & & \\
32 & & >16 & & & & & \\
 & >32 & & & & & & \\
64 & & & & & & & \\
\end{array}$$

. . . .

If x stands for any positive integer we have $\dfrac{2^{x+1}-2^x}{(x+1)-x}=2^x$. Using the notation $\Delta 2^x \equiv 2^{x+1}-2^x$ and $\Delta x \equiv (x+1)-x$ we get $\dfrac{\Delta 2^x}{\Delta x}=2^x$ in analogy to $\dfrac{de^x}{dx}=e^x$. The quality of reproducing itself which the exponential function $y=2^x$ has in regard to its differences when the exponent increases from 1 to 2 to 3 $\cdots$ reappears for the function $y=e^x$ in regard to the derivatives.

The relationship between the functions $y=2^x$ and $y=e^x$ can be illustrated through the following example in reference to a banking account. If a bank were to operate on a 100% compound interest rate, thus adding to the invested capital its own value as its interest after one year the investment of one dollar would be raised to two dollars at the end of the first year, to four dollars at the end of the second, to eight at the end of the third, etc. After n years we come to 2^n. If the bank were to use semi-annual compounding one-half of a year's interest would be added after the first half year. With the 100% interest rate, one-half dollar would be added after the first half year. During the second half of the year the capital would be $1+\frac{1}{2}$ and adding half of its value at the end of the second half year would bring it up to

$$(1+\tfrac{1}{2})+\tfrac{1}{2}(1+\tfrac{1}{2})=(1+\tfrac{1}{2})^2=1.5^2=2.25.$$

The original investment of one dollar will thus be raised to two dollars and twenty-five cents. Were the bank adding the interest to the capital after every one-third of a year, we would get at the end of the first year:

$$[(1+\tfrac{1}{3})+\tfrac{1}{3}(1+\tfrac{1}{3})]+\tfrac{1}{3}[(1+\tfrac{1}{3})+\tfrac{1}{3}(1+\tfrac{1}{3})]$$
$$=(1+\tfrac{1}{3})^3=2.37\cdots.$$

Similarly compounding after every quarter of the year would lead to

$$(1+\tfrac{1}{4})^4=2.44\cdots.$$

In general compounding n times per year, we would get from one dollar, at the end of the year $\left(1+\dfrac{1}{n}\right)^n$ and if n approaches infinity we come to $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n$ which is e. After x years with annual compounding we would obtain 2^x with continued compounding e^x. The process illustrated in reference to a bank account can be applied to any form of a growth and e can thus be considered as the value of 2 increased through a transformation of the repeated duplication into a continued process of growth.

If we then consider an entity a involved in a growth in which an increment from step to step is proportional to the value reached at the respective steps, k being a constant factor, we obtain:

beginning	a		
after the first step	$a+ka$		$=a(1+k)$
after the second step	$a(1+k)$	$+ka(1+k)$	$=a(1+k)^2$
after the third step	$a(1+k)^2$	$+ka(1+k)^2$	$=a(1+k)^3$
. . . .	. . . . . .	. . . . . .	. . . . . .
after the nth step	$a(1+k)^{n-1}$	$+ka(1+k)^{n-1}$	$=a(1+k)^n$.

The result is a geometric progression with the ratio $1+k$. Almost any natural growth follows this same principle. The larger the population of a town has become, the larger will be the additional population per annum (under normal conditions), the larger an institution has become, the larger will be its further growth if the conditions continue which prevailed during its development. The closest possible relation between the increment and the growing entity prevails when $k=1$, that is when the increment equals the growing entity. In this case we get a geometric progression with the ratio 2, and if the process develops into a continued growth, 2 is replaced by e. The term "natural" in regard to e is justified.

The relationship between e and the sequence 1; 2; 4; 8; $\cdots$ comes to the fore in the development of e as a continued fraction. Whereas e written as a decimal fraction shows no regularity in the sequence of its digits, it does present as a continued fraction the following regular form:

$$e=1+1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{1+\cdots}}}}}}}}}.$$

e can be derived also from a geometric viewpoint. In Figure 1 a circle has been divided into twelve equal parts. Each point of division has been joined with the center of the circle so that twelve radii were obtained. From the end of one of the radii (the one vertically upwards) a perpendicular has been drawn to the next radius. From its footpoint another perpendicular has been added to the following radius etc. The consecutive footpoints lie on a logarithmic spiral. The ratios of the distances from the center of the circle to

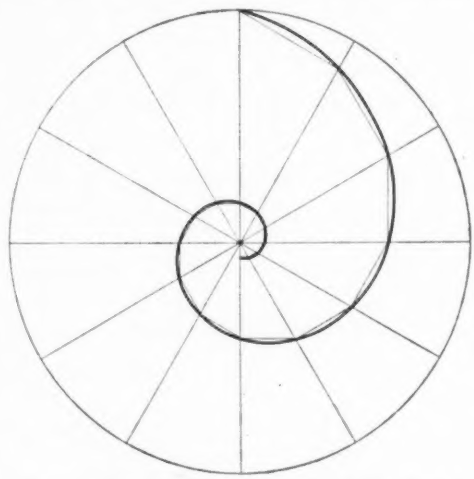

FIG. 1. Construction of a logarithmic spiral.

the points of the logarithmic spiral taken along two consecutive radii are

$$\cos\frac{360°}{12}=\cos 30°.$$

If r stands for the radius of a circle, the logarithmic spiral will arrive after its radius has turned 360° around the center, at a distance of $r\cdot(\cos 30°)^{12}=r\cdot 0.1778$ from the center. For every choice made in respect to the amount of points of division along the circle the construction will lead to a particular form of a logarithmic spiral. These spirals move faster to the center if

there are fewer points of division and slower if there are more.

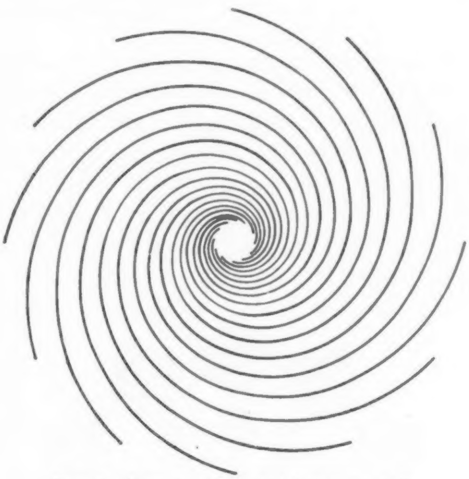

FIG. 2. Family of logarithmic spirals.

In Figure 2 logarithmic spirals have been drawn along the same lines as in Figure 1 but here at every one of the twelve points of division along the circle, a logarithmic spiral has been started; the construction lines are omitted.

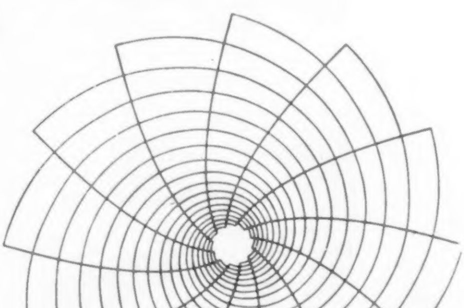

FIG. 3. Family of logarithmic spirals and orthogonal trajectories.

Figure 3 contains the same family of logarithmic spirals as Figure 2, but to them have been added the orthogonal trajectories (curves intersecting the family of curves at right angles). Those are again logarithmic spirals but of a different form. They move very much faster towards the center than the spirals of the original family. Changing of the original family of logarithmic spirals, by changing in their construction the number of points of division along the circle would also change the orthogonal trajectories but in an opposite manner. If the original spirals were made to lead more straightly at the center the orthogonal trajectories would become rounder and vice versa. Therefore one particular family of logarithmic spirals will exist which will have the same form as their orthogonal trajectories. This particular family of logarithmic spirals is shown in Figure 4.

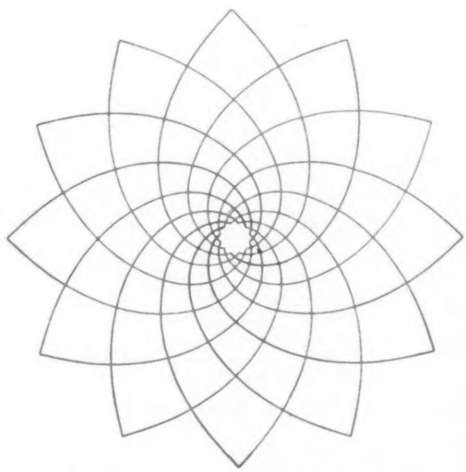

FIG. 4. Logarithmic spirals which are of the same form as their orthogonal trajectories.

The equation of this particular family of logarithmic spirals has to satisfy the condition that its basic ratio (between radii forming a given angle) is the same as the one of the orthogonal trajectories, both families considered rotating in opposite directions. In polar coordinates the equation of a logarithmic spiral with any ratio a between two radii forming an angle of one radian (57.29°, the angle for which the arc equals the radius) is $\rho = a^{\theta + c}$, ρ being the radius vector, θ the vectorial angle measured in radians and c the arbitrary parameter of the family of curves determining the position of a single spiral in

regard to the polar axis. Differentiating $\rho=a^{\theta+c}$ to come to the differential equation of the family of spirals gives $\frac{d\rho}{d\theta}=a^{\theta+c}\cdot\ln a$. Eliminating c by substituting $a^{\theta+c}=\rho$ gives $\frac{d\rho}{d\theta}=\rho\cdot\ln a$. In order to come to the differential equation of the family of orthogonal trajectories $\frac{d\rho}{d\theta}$ is replaced by $-\rho^2\cdot\frac{d\theta}{d\rho}$ and we obtain $-\rho^2\cdot\frac{d\theta}{d\rho}=\rho\cdot\ln a$ or $-\rho\frac{d\theta}{d\rho}=\ln a$. Solving the differential equation by separating the variables we obtain

$$-d\theta=\ln a\cdot\frac{d\rho}{\rho}$$

and through integration:

$$-\theta=\ln a\cdot\ln\rho+c_1;$$

c_1 being the constant of integration. Solving for ρ we get

$$\ln a\cdot\ln\rho=-\theta-c_1$$

$$\ln\rho=\frac{1}{\ln a}(-\theta-c_1)$$

$$\rho=e^{1/\ln a\cdot(-\theta-c_1)}=e^{-1/(\ln a)\theta-c_1/(\ln a)}$$

and for $e^{-c_1/\ln a}\equiv c_2$, we obtain

$$\rho=e^{-1/(\ln a)\theta+c_2}.$$

The result is the equation of another family of logarithmic spirals. Writing for the equation of the original spirals instead of $\rho=a^{\theta+c}$ the expression

$$\rho=(e^{\ln a})^{\theta+c}=e^{\ln a\cdot\theta+\ln a\cdot c}$$

and for $e^{\ln a\cdot c}\equiv c_3$ we get $\rho=e^{\ln a\theta+c_3}$. The condition that both families have the same ratio but turn in the opposite sense (negative sign for θ) is $\ln a=\frac{1}{\ln a}$ or $(\ln a)^2=1$; $\ln a=\pm 1$. Therefore $e^{\ln a}=e^{\pm 1}$ and $a=e^{\pm 1}$. The first solution is $a=e$ and the second $a=\frac{1}{e}\cdot$ Therefore the particular family of logarithmic spirals has the equation $\rho=e^{\theta+c_3}$ or $\rho=e^{-\theta+c_3}$. For the corresponding families of orthogonal trajectories we obtain by substituting the values $\ln a=\pm 1$:

$$\rho=e^{-\theta+c_2}\quad\text{and}\quad\rho=e^{\theta+c_2}.$$

The two solutions express a reciprocal relationship between the two families of logarithmic spirals; if one represents the original family the other represents the orthogonal trajectories and vice versa. We therefore have only one pair of families of logarithmic spirals which are orthogonal trajectories.

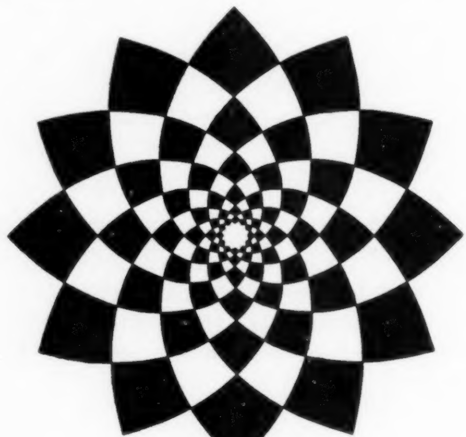

Fig. 5

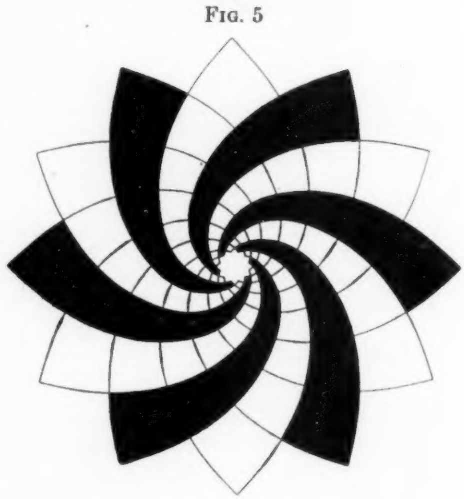

Fig. 6

In the equation of the logarithmic spirals of Figure 4: $\rho=e^{\theta+c_3}$ and $\rho=e^{-\theta+c_3}$ e takes the place of a. If we take two radii separated by an angle of 1 radian their

lengths are $\rho_1=e^{\theta+c_3}$ and $\rho_2=e^{\theta+1+c_3}$ and their ratio is

$$\frac{\rho_2}{\rho_1}=\frac{e^{\theta+1+c_3}}{e^{\theta+c_3}}=e.$$

The particular system of logarithmic spirals therefore contains the ratio e between any two radii which form an angle of one radian.

Logarithmic spirals being an adequate expression of an organic growth, of an evolutionary principle, have inspired the great mathematician Jacques Bernoulli to order a logarithmic spiral to be engraved on his tombstone with the inscription: "Eadem mutato resurgo (In different form, yet still the same, shall I arise)." Among all possible logarithmic spirals only one form has the quality of balance to be congruent to its orthogonal trajectories and this one is based on the ratio e. Taken as a form this logarithmic spiral expresses a specific harmony.

The three figures 5–7 (pp. 354–355) contain the same families of curves as Figure 4, the logarithmic spirals based upon e, but they emphasize different elements in order to bring out various characteristics. In Figure 5 the areas between the logarithmic spirals are shown alternately in black and white in a checkerboard manner. In Figure 6 the areas between one system of logarithmic spirals are alternately black and white which underlines the spiral form. In Figure 7 the distribution of the black and white areas forms an axis or symmetry and brings out the mutual relation of the orthogonal trajectories. The diagrams presenting the outstanding systems of logarithmic spirals based upon e may also be considered among the many contributions of mathematics to the domain of the arts.

FIG. 7

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Problems, Problems Everywhere

By Valine Hobbs
Demonstration School, Stephen F. Austin State Teachers College, Nacogdoches, Texas

Arithmetic, like gold, is where you find it. In order to vitalize problem solving for children in the fifth and sixth grades, I ask my pupils to find real problems at home and at school. This plan has many advantages, the first being the recognition of a problem. One of the weaker students brought this for a problem: "We have eighteen windows and twelve chairs. How many more windows than chairs have we?" After considerable discussion, the group voted that one out as having no point. What difference does it make if one has more chairs or more windows? They are not things that can be compared. Another child read, "I have eighteen hens and got fourteen eggs one day. How many eggs and hens have I?" A brief discussion brought out that the problem there was not how many eggs and hens one had, but "How many hens did not lay that day?" an item of importance to any serious chicken raiser.

Clear and complete details must be told for correct solving of problems. The little girl who brought this problem, "We went to Houston, 140 miles, on half a tank of gas. How much did we average per gallon?" was met with a barrage of "How many gallons does your tank hold?" and saw at once that she had omitted an important part of the problem.

Some of the problems are humorously revealing, for example: "Daddy went hunting with 24 shells and came back with 4 ducks. How many shells did he average per duck?" Evidently Daddy is not such a hot shot!

Such problem writing gives unlimited opportunities for the teaching of correct English. Many children (and adults) say *like* instead of *lack*, but this misuse lessens when the children write as well as read the problems in which the word is used. Other correct forms which such problem writing helps to make habitual are "How many are," "How many were"; "There are," "There were"; the use of the comparative instead of the superlative in comparing only two things; spelling a number at the beginning of a sentence; the correct use of the dollar mark, decimal, and cent sign; the use of the hyphen in certain numbers; correct paragraphing and indentation; comma in a series; capitalization; periods; question marks; abbreviations; and correct spelling.

Not only must the writing of the problems be correct, but the reading must be clear because the hearers must understand what is said before solving is possible. In addition, such homespun problems give a teacher better insight into the children's interests and their homelife.

All in all, our problems have been very interesting and we plan to save them and make our own arithmetic book. Here are some of the most representative ones found at home and at school. Some may seem too easy for children of this grade level, but they represent the ability of the children who brought them.

1. Everybody in our room pays $5 a semester for tuition. How much will be paid by the 15 children in the fifth grade? How much will be paid by the 16 children in the sixth grade? How much will be paid by the entire room?

2. We pay fifty cents for a year's subscription to *My Weekly Reader*. How much will our teacher send away with the order? (That one always catches somebody—Teacher buys a paper too!)

3. I have a World War picture book of

588 pages. I am on page 223. How many more pages have I to look at? (There are always many problems similar to this one in regard to reading books.)

4. Mother had the Auxiliary to meet with her on Thursday. Nine members came and each ate two doughnuts. How many were left of the two dozen that Mother bought?

5. My sister and I have 105 toys on the three shelves of our toy cabinet. What is the average number per shelf?

6. There are 156 books on the three shelves in our livingroom. What is the average number per shelf? (Children love to find averages. I do not know why unless it is because they have heard their parents use the term without explaining it. They also like problems about shelves, probably because shelves are connected with their home and school chores.)

7. My piano has 52 white keys and 36 black ones. How many keys are there in all? How many more white keys than black ones are there? How many octaves are on a piano? (That one almost caused a riot because some of the children not familiar with a piano did not know what an octave was and argued that the question was not fair. Their objection was sustained and the music teacher gave a drill on octaves.)

8. Mother's grocery bill for week before last was $21.44. For last week it was only $16.24. How much less was it last week than the week before? (I almost wept over that one for I knew what a hard time that family was having.)

9. I have one puzzle with 1,000 pieces and another with 375 pieces. How many more pieces are in the first puzzle than in the second?

10. Yesterday I went to a movie. I had fifty cents. I spent nine cents for my ticket and five cents for some mints. How much had I left?

11. We have 15 cows and 2 mules on our farm. How many animals have we? (This one was from a very backward child who was sometimes omitted from activities because he was so slow. He was delighted to have a problem that all the others could solve and basked in the resulting popularity!)

12. I helped Mother wash windows. I washed 2 in one room, 6 in another, and 4 in another. How many windows did I wash? Each window has four panes. How many panes did I wash?

13. We have planted 17 acres in cotton, 18 in corn, 6 in peanuts, 1 in watermelons, 2 in peas, and 2 in potatoes. How many acres have we planted in all? (This problem came from a shy litt'e farm boy who had said the day before that he could not find many problems at home because they did not buy many things.)

14. I always dust the chairs for Mother. There are 4 in the livingroom, 6 in the diningroom, 2 in my room, 3 in Mother's room, 1 in the guestroom, and 2 on the porch. How many chairs do I dust?

15. I help to feed 5 horses, 9 mules, 2 cows, 12 little pigs, and 5 hogs every night. How many animals do we feed?

16. Thirteen other Brownies and I went on a hike. Daddy gave us a box of cookies from his store. The box had 7 rows of cakes with 12 in a row. How many cookies were there? How many did each Brownie have?

17. We get two newspapers a day. How many do we get in a year? (Children like this type, which they call a one-number problem.)

18. In one of the Conference games our men weighed as follows: 200 lbs., 183 lbs., 195 lbs., 200 lbs., 205 lbs., 183 lbs., 191 lbs., 185 lbs., 160 lbs., 175 lbs., 200 lbs. What was their average weight?

19. Marshall lost the bi-district game to the Panthers, 32 to 6. By how many points was Marshall outscored?

20. We are knitting an afghan as a Junior Red Cross activity. Each block is six inches square. How many square inches are in each block?

The afghan will be 8 blocks wide and 12 blocks long. How many blocks will there be in the afghan?

How many feet wide will the afghan be?

How many feet long will it be? How many square feet will it contain?

We want 96 squares for our afghan. We now have ——. How many more squares do we need? (A daily problem until the afghan was finished.)

Each square weighs approximately one ounce. About how much will the finished afghan weigh?

21. Mother made 25 pounds of fruit cake. If she had bought the cake, it would have cost at least 45¢ a pound. How much would that have been?

We found that the materials for the cakes cost only $5.50. How much did Mother save by making the cakes at home?

22. I collect stamps. I have 28 American stamps, 7 Argentinian, 4 Canadian, 4 Chinese, 13 German, 3 Swedish, 9 Japanese, 3 Indian, and 3 Costa Rican. How many stamps are in my album? (Oh, the struggle with the spelling of those hard words!)

23. Our milk bill for last month was $12.15. There are five in our family. What was the average cost per person? What was the average cost per person per day for milk?

24. Mother bought 3 yards of velveteen for my new dress. It cost $2 a yard and the buttons cost $1. How much did my dress cost?

It was just as pretty as a ready-made one that I tried on that cost $12.95. How much did we save by making the dress?

25. Our side porch has been glassed in. Now we want to buy linoleum for it. It is $11\frac{1}{2}$ feet wide and $28\frac{1}{2}$ feet long. How much linoleum will it take? (This was one of the best problems brought in because it involved art as well as arithmetic. The children finally decided that the thing to do was to buy two strips the length of the porch and make a seam in the center, most linoleum being six feet wide. A few inches down each side would be wasted, but the floor would look better that way.)

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THE general educator is usually found to concede that mathematics should be taught in our high schools, but he is frequently heard to assert that it should be elective. Many teachers of mathematics, perhaps most of them, would personally welcome such a chance, since the pleasure of teaching it largely increased if the learner takes the subject *con amore*. But of late a new type of educator has appeared, the one who proposes to weigh in psychological scale the intellect of youth and to guide it aright. You have it in your own part of the country today in the phase of vocational guidance, and in this work so many excellent people are seriously engaged that we are certain to see it become an important phase of modern education. Let the boy who gives promise in science begin his specialization early, say those who seek to guide the youth in a scientific manner, and let the one who takes to Latin bend his energies there. Let there be scientific tests to show whether or not the particular individual can hope for success in the particular vocation—a worthy effort and one that will produce good results. But there are not wanting those who will be less scientific, and who will sssert that one who, by virtue of his surroundings and family, is destined to be a hewer of wood, should early come to like to hew, and should be taught chiefly the nobility of labor with the hand. That we may realize some of the dangers that beset those who seek to guide the youth aright, and who may feel called upon to sidetrack all that is not immediately practical, let me tell you some advice that I myself have given within a few years past in cases like these, and lay before you the problem that I had to face.

Now long ago there came to me a father who wished to train his boy for trade in a seaport town, and who asked my advice as to the proper education to give him. The problem seemed simple. The community was not an educated one; it lived off its little shipping industry; the boy was destined to small business and to small reward; he gave no promise of anything better, and the advice was, therefore, unhesitatingly offered that the only mathematics he needed was arithmetic through the sixth grade.

Another parent asked me a little later about his son. The boy was of the ordinary type and would probably follow his father's occupation, that of a sculptor. What mathematics would it be well for him to take? I suggested a little study of curves, some geometric drawing, and the modeling of the common solids—a bit of vocational guidance that seemed to me then and seems even yet particularly happy.

A third boy happend to be with me on a steamer and I took some interest in talking with him and with his mother. They lived in a city of no particular note, at any rate at that time, and the boy was going into the selling of oil within a few years. The profits of the Standard Oil Company appealed to the family, and I advised him to learn his arithmetic well and get into business as soon as he could.

Out of the store of my memory I recall a curious lad whom I came to know through my sympathy with the family. The mother was a poor woman, and she took the boy, when little more than a baby, over to Riverside Park one day when there was a naval parade. A drunken sailor, having had a fight with a group of hoodlums, rushed through the crowd of spec-

* The following is a significant part of an address on "Problems in the Teaching of Secondary Mathematics" delivered before the New England Association of Teachers of Mathematics many years ago by Professor David Eugene Smith. It has a modern ring and shows the way in which Professor Smith was able to present important matters concerning Mathematical Education to teachers in that field. Reprinted by permission from a pamphlet published by Ginn and Co.—Editor.

tators and slashed right and left with a knife. In the excitement the boy, in his mother's arms, was horribly cut in the face. When I knew them he was about ten years old, unable to speak plainly, and already a misanthrope through his affliction. I advised the mother to give the boy a vocational education, telling her that through the use of the hands he would satisfy his desire for motor activity, and that this would compensate him for the loss of verbal fluency and would tend to make him more contented with his lot. In this advice I feel that I would have the approval of educational circles.

And finally, out of this series of experiences, let me recall the case of a boy whom I came to know through a noble priest who found him one morning, an infant a few days old, on the steps of his church. We talked over the best thing to do for such a foundling, one who, at the time I I knew him, was in the primary grades. He showed no great promise, he was without family recognition, and his only chance, apparently, was in the humbler walks of life. I recommended a vocational school where he could quickly prepare for the shop or the lower positions of trade, and the good priest approved my plan at the time, although he finally followed quite a different course.

It is apparent, however, that I have here spoken in parables. Perhaps you already recognize the boys, and perhaps you feel how sadly I blundered in my counsel. For the first of these whose cases I have set before you felt a surging of the soul a little later, and this was recognized in time, and he became one of the Seven Wise Men of Greece—Thales the philosopher, he who introduced the scientific study of geometry into Greece. The second felt a similar struggle of the soul, and his parents recognized my poor counsel in time to save him and to give to the world the founder of its first university—Pythagoras of Samos. The third boy, for whom only the path of commerce seemed open, and this in a town only just beginning to be known, was the man who finally set the world's first college-entrance examination, the one who wrote over the portal of the grove of Academos the words, "Let no one ignorant of geometry enter here"—Plato, the greatest thinker of all antiquity. The fourth, the hopeless son of poverty, maimed, sickly, with no chance beyond that of laboring in the shop for such wage as might by good fortune fall to his lot, became the greatest mathematician of his day—always the stammerer (Tartaglia), but one whom Italy has delighted to honor for more than three centuries. And the last one of the list, the poor foundling on the steps of St. Jean-le-Rond in Paris, became D'Alembert, one of the greatest mathematicians that France, a mother of mathematicians, ever produced.

Shall we, then, advocate the selection of those who are to study mathematics and close the door to all the rest? Are we so wise that we can foresee the one who is to like the subject, or succeed in it? Have we so adjusted the scales of psychology that we can weigh the creases in the brain or is there yet invented an X-ray that will reveal to us the fashioning of the cells that make up its convolutions?

Of course it will at once be said that these illustrations that I have given are interesting, but that they are unfairly selected; that those boys gave earlier promise in mathematics than I have said. It will be asserted that I should have taken the casė of the stupid boy the one who did not like school, the one who liked to play with little wind wheels, who liked to fight, who actually did run away from school, and who stood near the bottom of the class in mathematics. Such a case would be a fair one, one in which we could safely say that prescribed algebra and geometry are out of place. And I suppose we must agree to this and confess that the argument from the historical incidents that I have mentioned was unsound. Let us rather take this case that I have just described, and let us see to whom the description applies. I need hardly tell you

who this boy is; he is well known to you; he is well known to the world; and long after every educational reformer has passed into oblivion his name will stand forth as one of England's greatest treasures, for it is the name of Sir Isaac Newton.

But again I have been unfair, perhaps. I should have taken positively hopeless cases, for such can surely be found. I should have taken some illiterate man, one who does not learn to read until he is nearly out of his teens, or else some man who shows no promise in mathematics by the time he reaches manhood, or some one who by the time he is thirty is to show no aptitude in the science. It is so easy to theorize! But let us have care, for the men whom I have now described are Einstein, Boole, and Fermat. Take them away and where is your theory of invariants, your modern logic of mathematics, and the greatest genius in theory of numbers that the world has ever seen?

But I am wandering afield, and I fear I may be interpreted to question the modernizing of our educational work. I thoroughly favor scientific, vocational guidance as undertaken by a small number of our best scholars in the field. With equal zeal do I favor industrial education when it is not so narrow as to condemn a boy to some particular groove in life, and I earnestly hope that we shall so guide our youth that every boy and girl will leave school fitted to do something well. I do not believe that any thoughtful educator wishes to guide a pupil in a narrow path nor keep from him the chance that the world owes him. It seems right, however, to set the problem clearly before us: Can we safely say that we may close the door of mathematics to any boy? Should he not be given the chance? If he fails, that ends it, but if he succeeds, the world is the winner in the lottery. Of course this does not answer the question as to what this chance should be; it is quite possible that it should not be our present algebra; it is even possible that it should be merely some form of mensuration that masks under the time-honored name of geometry; it may conceivably be some emasculated form of fused mathematics that has none of the logic of geometry and none of the beauty of algebra, although I do not believe it; and it may even be some form of technical shop mathematics that appeals to but few pupils because of its very technicalities. This is the part of the problem to be solved. But that the door of mathematics of some substantial character shall not be opened, and opened after arithmetic has been laid aside as the leading topic, seems unthinkable.

A Study in Human Stupidity

By Mary Ann Woodard
South Orange, N. J.

To alleviate the monotony of drill on algebraic fundamentals Gustav Davidson's story of Galois' life was read to the class. The suggestion that it was excellent material for dramatization was enthusiastically received. The admonition that any production must be kept simple so that a minimum of time would be spent in memorizing and staging was given. The following was the result.

The curtain announcement was adapted from Lillian Lieber's "Galois and the Theory of Groups," page one.

The particular story
Treated in this little play
Is
The life of
Evariste Galois.

"Galois died,
Just one hundred years ago,
Before he reached the age of
Twenty-one!
In his short and tragic life
He developed
A branch of mathematics,
Which is of the greatest importance
Today.

He is ranked among the
Twenty-five greatest mathematicians
That EVER lived.*

Outside of his tremendous success
In his mathematical work,
His life was a series of
Frustrations."

We present this play with the hope that you will enjoy it and learn a little of the history of mathematics.

* G. A. Miller in *Science*, Jan. 22, 1932.

Written, directed and produced by Thomas Casserly, Jr., II, *and* Bernard Devin, Jr., *South Orange Junior High School, South Orange, New Jersey. Presented March, 1943, under the sponsorship of* Miss Mary Ann Woodard.

A biographical play on the life of the French mathematician Evariste Galois.

Reference: The Life of Evariste Galois; Gustav Davidson: Scripta Mathematica reprint. Galois and the Theory of Groups; L. Lieber, Science Printing Press, Lancaster, Pa., 1932. Men of Mathematics; Eric T. Bell, Simon and Schuster, New York, 1937.

Characters in order of appearance:

Narrator
Peasant
M. Jordan
First Professor
Second Professor
Third Professor
Secretary of the École Polytechnique
Secretary of the Academy of Sciences
Head of École Normale
Messenger
Spectator
Evariste Galois
Spokesman of the Academy of Sciences
First Patriot

Second Patriot
Guard
Spectators Patriots

The play is intended for presentation with the aid of a small public address system, giving the narration a proper dramatic impact, and allowing the use of recorded background music. The first movement of Tschaikovsky's Fourth Symphony is suggested.

Houselight off. Very quiet. Curtain is closed. Music begins, reaches introductory climax, and dies.

NARRATOR: A body? A body in the road? What did it matter? Times were hard. What was it the authorities said—shot through the abdomen. No—not the first victim of a duel the province of Gentilly had seen by eighteen hundred thirty-two.

Curtain opens quickly but softly.

(*Stage is lit dimly to preserve dramatic atmosphere, to avoid need of elaborate set, and to simulate early dawn. Body is lying in heap in near center. Peasant enters from right. Carries apparently very heavy bag of produce. Gets nearly on top of body before noticing it.*)

PEASANT: (*Allowing bag to slip toward ground*) LORD! Again! (*advances to and bends over body being careful to speak toward audience*) Poor chap's been shot. No one I know. Wait! He's breathing. Perhaps there is still time. The Cochin Hospital is not far. (*Looks sympathetically at body*) So young—so frail looking. But come—there is no time to waste. (*Straightens*) .

Lights go out. Then curtain closes.

NARRATOR: (*Again soft music*) Of course the fellow died. Peasants of France have no knowledge of first aid. And so on the thirtieth day of May, Anno Domini 1832, Evariste Galois passed on to a better life. He was so young—only twenty, you know, so—but—that wouldn't interest you. It didn't interest anyone else. They all laughed when you mentioned his name. Yes, every last fool and simpleton and ignoramus laughed and tapped his forehead significantly. Let us turn from this sad situation to the study of a scholar, a generation later. M. Jordan, a mathematician by trade sits pouring over ponderous volumes and piles of paper.

(*Curtain opens disclosing a middle aged man seated at a desk. The stage is softly illumined. He writes as the narrator continues.*)

Let us just get a little closer—what is that I see scribbled at the end of all those symbols? Why it is the name, Evariste Galois.

(*Pause for effect. Music does not swell.*)

Someone is approaching the desk. It is a small child.

CHILD: Papa I . . .

M. JORDAN: (*Very irritably*) Go away Lorraine! Can't you see I'm busy?

CHILD: Yes sir.

M. JORDAN: Lorraine, I am sorry. I really must learn to control my temper. But these accursed figures!

CHILD: What are they Papa?

M. JORDAN: Nothing, child. Go to bed—and say goodnight to your mother for me.

(*Exit child, Jordan returns to his work.*)

NARRATOR: (*again soft music*) We may well excuse M. Jordan for his outburst. For he is in the process of comprehending the scientific papers of one of the world's greatest mathematicians, Evariste Galois. Do you wish to know more? It is a sad story.

(*Pause.*)

Evariste Galois' short life began in eighteen hundred eleven. At twelve his parents sent him to the college of Lois Le Grand in Paris. It was not Evariste's fault that he was mathematically precocious, that he knew more than those who taught him. Resenting his genius his professors made no move to further it.

(*First and Second Professors enter apron in front of curtain from the left. May be spotlighted.*)

FIRST PROFESSOR: Why only today he questioned what I lectured in class. He spends five minutes on a two hour assignment.

SECOND PROFESSOR: But does he arrive at the correct solutions to your assignments?

FIRST PROFESSOR: Yes—but it is not right. He spends all his spare time reading books that are beyond him. Why *even* I would have to study them for hours to understand their true import. He is a queer insubordinate boy.

(*They meet Third Professor at center—talk quietly among themselves.*)

NARRATOR: This same criminal indifference and neglect greeted young Galois in everything he did. At sixteen Galois was absorbed in equations of the fifth degree, vainly hoping to prove them soluble. Though he nor anyone else ever accomplished this, his research opened the way to present day methods of solving equations with algebra.

(*Pause.*)

And still they laughed.

THIRD PROFESSOR: (*With pomp and authoritative finality*) His ability in which one is supposed to believe, but of which I have not witnessed a single proof, will lead him nowhere.

(*Three Professors leave at right.*)

NARRATOR: At seventeen Galois decided to enter the École Polytechnique.

(*Spot shows secretary seated at desk in front of curtain.*)

SECRETARY OF ÉCOLE POLYTECHNIQUE: (*Writes as he speaks in broken phrases*) We regret to inform you—that after due consideration of your—application—we can find no place for you in our—institution.

(*Spot out. Music swells, then softens.*)

NARRATOR: Undiscouraged Evariste continued his attempts to enter a mathematical school. Meanwhile, convinced that he had discovered a number o truths of a fundamental and far reaching nature, he submitted a memoir to the Academy of Sciences. No answer came from the academy, not even an acknowledgment. At length Galois wrote to learn the fate of his paper.

(*Voice from left exit near stage.*)

SECRETARY OF THE ACADEMY OF SCIENCES: We are sorry to inform you that the paper of which you write has been mislaid. Yours very truly ———.

(*Music comes out and drowns out last.*)

NARRATOR: (*Music softens*) Do you know what happened to the Paper? (*Intensely*) It was lost—lost through the infernal numbheadedness of Cauchy, a mathematician greatly admired by Galois in his youth. (*Bitterly*) Ah paradox!

(*Pause.*)

Continuing his efforts the boy finally entered the École Normale, a fairly good school. Managed to enter. Indeed, he who knew more than his faculty, who could run rings around his professors—(*sarcastically*) they very kindly saw fit to accept him in their little kindergarten when they should have considered it an honor for him even to know the place existed.

For two semesters Evariste remained at École Normale, a time spent in constant battle for recognition from the blundering, know-it-all, cruelly indifferent professors. The inertia of their minds could not conceive of his genius and the miracles he was begging them to accept.

Sensitive, impatient of delay and the dull-wittedness of those above him, the boy became more and more introspective. Then at the conclusion of the second semester. . . .

(Spot shows man seated at desk. Galois stands just out of the area of light, a thin, tall, almost pathetic figure.)

HEAD OF ÉCOLE NORMALE: Galois, we have discussed your case and have come to the conclusion that your presence here has a detrimental effect upon the student body. Therefore we are forced to order your dismissal.

(Spot out.)

NARRATOR: And as the last drop in Evariste's cup of despair . . .

(Voice through microphone.)

MESSENGER: M. Galois? I have been sent to inform you that your father has just committed suicide.

(Music swells, then softens.)

NARRATOR: Expelled from school, virtually destitute, Evariste sought to sustain himself by delivering lectures in the rear of a bookshop run by a man named Calliot. Out of pity or curiosity there were always listeners. They certainly did not—could not—comprehend him. For Galois was now in the highest realms of mathematical theory, treading those mystic heights where only the initiated may follow. The truth of the matter is that the whole mathematical faculty of the École Polytechnique would have come away from the little bookshop with exactly the same comprehension as did the loafers who listened.

(Curtain opens disclosing interior of bookshop. Person playing Galois must be convincing in his portrayal of a discouraged, emaciated youth who though he has been thwarted at every turn, is very eager—just a trifle fanatically overeager, to impart some of his genius to others. The details of the interior are simple. A few tables, a number of books and perhaps a chair will suffice. Grouping of listeners is up to director. A "crowd" of about ten persons comprises this group.)

EVARISTE GALOIS: *(As if continuing a lengthy exposition) (Pauses to wait for effect of statement. Group makes no sound)* Perhaps if I put it in a simpler way. Given: A group, G, with n elements in it: and a subgroup, H. Containing r elements. To show that r is a factor of n. Let the elements of H be:

$$a_1, a_2, a_3, \cdots a_r.$$

Now choose some element, b, in G but not in H, and multiply it by each of the r elements, etc.*

FIRST SPECTATOR: M. Galois, I fear the hour is late. Perhaps another time . . . .

GALOIS: *(sighs)* You are right my friend. Thank you for listening. Good night.

(Group files out with general goodbyes and shrugging of shoulders. Calliot, the owner of the shop, and Evariste remain.

CALLIOT: You did well today my friend. While they were here they bought eight books.

GALOIS: M. Calliot! I pour out my heart and soul to them in an earnest endeavor to create some glimmer of light in their feeble intellects and you—you worry about your book sales!!

* L. Lieber, "Galois and the Theory of Groups" cover.

CALLIOT: (*Eager to prevent Galois from becoming angry*) Well one must eat you know.

GALOIS: Do I know!! Man—would I stand here day after day haranguing people like a fishwife, were that not too true? Nay—I would be hard at work proving my theories until I should be recognized. (*Dreamily*) Someday, perhaps, that day will come. Hah! That is poetic, is it not! Calliot—tomorrow I shall send another memoir to the Academy of Sciences. Baron Fourier is now secretary. Perhaps he will listen to me.

(*Lights out. Music swells, Curtain closes.*)

NARRATOR: Perhaps, at last, Evariste will get some results. For Baron Fourier has taken this second memoir home with him to read. Surely he will see that, here, mathematical conclusions of the sublimest order are set down. Surely he—wait! What is this? The Baron's home is draped with funeral wreathes. Surely—yes—the Baron is dead. (*Pause—then eagerly*) It is possible he did read the memoir. But no—he didn't. (*Bitterly*) Listen to the logic of this spokesman for the Academy.

SPOKESMAN: The loss is a very simple thing. Baron Fourier was to have read it, but d'ed, and so, the memoir got lost. (*This from right front exit near stage*)

NARRATOR: Evariste was then ready to give up. But he met another mathematician a man named Poisson. Poisson offered to intercede for Galois at the Academy. Once again the memoir was written and intrusted to Poisson. But Poisson was in no hurry. For four long months the memoir lay on his desk. After a number of reminders from Galois the memoir was returned marked "incomprehensible." This was indeed the last straw. Filled with the fury of broken hopes, Evariste threw himself headlong into the political maelstrom of the French Revolution. At a meeting of patriots came Galois' final undoing.

(*Curtain opens disclosing group of patriots, about ten in number, around a table, as if finishing a banquet. Everyone is quite happpy and, as the curtain opens, a buzz of conversation is heard. After a momen' Galois rises from his chair holding a wine glass aloft in his right hand and in his left a small dagger or pocket knife.*)

EVARISTE GALOIS: (*Unconsciously flourishing knife*) A toast gentlemen. To Louis Phillipe! May his life be long! (*Galois in his excitement is thus deliberately p oposing a toast to the bitter enemy of his group. However, instead of laughing at the joke, the group misinterprets the significance of the waved dagger*)

FIRST PATRIOT: Hah! Evariste Galois, you are clever! You wave the knife, eh? And on what date do you intend to do away with Louis Phillipe?

GALOIS: (*aghast*) Do away with Louis Phillipe? Clever with the knife? Why I was using it to cut my chicken. I never. . . .

SECOND PATRIOT: That is all right, Evariste. We have no spies. Why are you so afraid to admit. . . .

(*Enter guard, hurriedly.*)

GUARD: Quick—Soldiers outside—they have heard everything. To the secret passage!!

(*Assemblage departs in wild confusion as the curtains close.*)

NARRATOR: Secret passages are not always sure avenues of escape and Galois was thrown into prison. Unable to present concrete evidence they had to free him. Two months later Galois was again locked up, this time as a dangerous agitator. Again he was freed, on parole.

But life no longer meant anything to him. Mathematical conclusions that could have electrified the sciences flashed through his brain, but now he shunned them. They had brought him nothing but grief and desolation. Evariste became reckless and defiant.

(*Pause.*)

And then he was embroiled by a woman. The exact circumstances are obscure. All that is known is that sometime on May 26 or 27 he was challenged to a duel by two men purporting to be the uncle and fiance of the woman, but who were her confederates. The circumstances were such that Galois could not refuse the challenge and keep his face, although he knew that acceptance meant certain death. On the eve of his tragic end he wrote in an open letter to all republicans . . .

(Footlights on. Galois stands sadly in front of curtains. Definitely no spot.)

GALOIS: "I die the victim of an infamous coquette. My life is quenched in a miserable piece of gossip. Forgive those who have brought death upon me; they are of good faith."*

(Lights out.)

NARRATOR: Evariste Galois also wrote another letter before he died. In it he gave a resume of all his contributions to mathematics. This letter he sent to his friend. Auguste Chavelier. That letter today, carefully preserved, is one of the most precious documents in science. Even today the full flowering of Galois' genius is not for us to attest. That privilege is left for a mind that is still to come, as transcendent as Galois' and also attuned to the murmurings of the infinite sea.

(Music swells)

(Loud report) *Sound effects.*

A body? A body in the road? What did it matter Times were hard. What was it the authorities said?—Shot through the abdomen. No, not the first victim of a duel the province of Gentilly had seen by eighteen hundred thirty-two.

(Music continues. Swell-climax.)

FINIS

* Galois's "Letter to All Republicans."

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Models of the Regular Polyhedrons

By R. F. Graesser
University of Arizona, Tucson, Arizona

Models of the regular polyhedrons are commonly made from pasteboard. Models of wood are not difficult to make and are much more durable. Their construction might serve as a project in manual training. The following directions are intended to aid the student who is not familiar with some of the metrical relations existing in these solids. Since it is desirable to have all five models of the same height, dimensions are expressed in terms of the height h, which may be varied at will. It is convenient to take h in centimeters; $h = 10$ centimeters is an appropriate value.

Tetrahedron: Make a regular triangular prism with the altitude equal to h, and each side of the base equal to $\frac{1}{2}\sqrt{6}h = 1.225h$. Trim off the edges around the upper base by cutting along the three planes determined by the center of the upper base and the three sides of the lower base.

Octahedron: Make a regular square prism with the altitude equal to $\sqrt{3}h = 1.732h$, and each side of the base equal to $\frac{1}{2}\sqrt{6}h = 1.225h$. After drawing the traces of the mid-section on the lateral faces, locate the centers of the two bases. Trim off the edges of the upper and the lower bases by cutting along the eight planes each determined by the center of a base and a side of the mid-section.

Dodecahedron: Make a cube each edge equal to $\frac{1}{2}\sqrt{10-2\sqrt{5}}h = 1.176h$. Draw in each face a line segment parallel to two sides of the face and bisected by the center of the face. The lengths of these line segments should be $\frac{1}{2}\sqrt{2}\sqrt{25-11\sqrt{5}}h = 0.449h$. The line segments in adjacent faces should be non-parallel, i.e., perpendicular skew lines. Trim the cube by cutting along the twelve planes each determined by an end of a line segment taken with the line segment in the adjacent face. The result will be a regular dodecahedron of height h.

Icosahedron: Proceed as for the dodecahedron except that the cube should have its edges equal to $\frac{1}{2}\sqrt{3}(\sqrt{5}-1)h = 1.070h$, and the line segments in the faces should be of length $\frac{1}{2}\sqrt{3}(3-\sqrt{5})h = 0.662h$. The student now has a solid with twelve pentagonal faces, but the faces are not regular pentagons. This solid must now be trimmed by cutting along the eight planes each determined by three ends of the original line segments that are closest to a vertex of the original cube. It is not necessary, however, to cut along the planes in this order. The result will be a regular icosahedron of height h. It is clear that other materials besides wood might be used in carrying out these directions, for instance, modeling clay.

Sheet celluloid may also be used to construct transparent and rather durable models of the regular solids, which may then be strung with threads to represent the axes of symmetry. Sheet celluloid may be purchased in different thicknesses at automobile accessory stores. The so-called diagrams, or nets, to be used in cutting pasteboard for the usual models, are to be found in most solid geometry textbooks. These should be drawn carefully in their proper sizes on sheets of paper. The lengths of the edges for models of heights h are repeated from the preceding directions as follows: for the tetrahedron, $1.225h$; for the octahedron, $1.225h$; for the dodecahedron, $0.449h$; and for the icosahedron, $0.662h$. For the six regular pentagons forming half of the net for the dodecahedron of side $0.449h$, draw a circle of radius h in which inscribe a regular pentagon of side $\frac{1}{2}\sqrt{2}\sqrt{5-\sqrt{5}}h = 1.176h$, or inscribe this pentagon by using the usual geometrical construction. By con-

necting alternate vertices of this pentagon, we obtain within it another, smaller regular pentagon, which is one of the faces of our dodecahedron. By connecting the alt rnate vertices of this inner pentagon and producing the lines until they meet the sides of the outer pentagon, we obtain five more pentagons congruent with and surrounding the inner one. This is half of the net, all that we really need. Fasten the sheet celluloid over the nets by using drafting tape or Scotch tape. Then scratch the celluloid along the lines of the net by using an awl or other sharp-pointed instrument. The celluloid will be found to break readily along these scratches. In order to fasten together the faces of the polyhedrons, Scotch tape may be used for the thinner celluloid. For thicker celluloid a cement made by dissolving scraps of celluloid in acetone should be used, or a similar ready-made cement may be purchased. The edges can be held together with the hands until the cement sets.

An axis of symmetry is line about which a polyhedron may be rotated into itself by turning it through an angle of less than 360°. A regular polyhedron has an axis of symmetry through the center of each face, through the mid-point of each edge, and through each vertex. Their number is always one greater than the number of edges. The tetrahedron has two different kinds of axes of symmetry; each of the other regular solids has three kinds. If the axes of symmetry are to be represented by threads, holes for these threads should be punched with an awl in the celluloid faces before these are fastened together. Also the sides of the faces should be slightly notched and the vertices slightly clipped off where these threads are to pass. Mercerized cotton thread that has been rubbed with bee's wax may be well used. The different kinds of axes may be distinguished by threads of different colors After the punched and notched faces of the model have been fastened together, a single length of thread may be used for all of the axes or for all of the axes of one k nd. Thread it through the model with a long needle or a fine wire, then bring it to the next hole by wrapping it around the outside of the model. The two ends can then be tied together to hold the thread taut. The holes are closed with the cement, thus sealing the thread in place. The unwanted thread wound about the outside of the model can be clipped off with scissors.

It is possible to construct models of the regular polyhedrons by representing their edges by wires soldered together. Doubled-pointed toothpicks stuck into small corks, and glued to hold them in place, may also be used. These are easily made but not very satisfactory models. By omitting one face of a pasteboard model, it may be filled with plaster of Paris thus making a plaster model. The inside of the pasteboard model should be greased before pouring in the plaster of Paris. It is obvious that the methods here suggested may be used to make models of other solids.

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THE ART OF TEACHING

Roots of Numbers

By RICHARD MORRIS
Highland Park, N. J.

THE WRITER recently listened to a talk on the topic, "Pitfalls in the Teaching of Mathematics." Among other items, the speaker used the following illustrative examples as the basis for some of his remarks: $[(-5^2]^{\frac{1}{2}}=?$, $\sqrt{(-5)^2}=?$, and the square root of $25=?$ In response to the speaker's call for answers, members of the audience offered the following: ± 5 and -5 for the first, -5 for the second, and ± 5 for the third. It was contended in the discussion that such problems, especially the first and the second, are apt to be confusing to the student when he reaches the subject of radicals. It seems to the writer that some reasonable and simple definitions and suggestions should clarify the atmosphere. But, of course, the student must actually learn these definitions and heed the suggestions.

In mathematics the word root may be and is taken in two senses. It would seem appropriate to consider one as arithmetical and the other as algebraic. In the arithmetical sense, a root of a given number is a number, which raised to the power indicated by the index of the root, will yield the given number. It is this sense with which the student first becomes familiar. For instance, the square root of 25 is 5; the cube root of $27=3$; the fourth root of 16 is 2. In the algebraic sense, the word root is applied to the numbers or quantities which satisfy an equation.

DEFINITIONS IN THE ARITHMETICAL SENSE

1. The nth root of a positive number is one of its n equal factors, n being a positive integer. Remarks: Just now it is not a question as to the form that root may take, nor whether that root may be obtained in a specified form, but it is important to accept the definition and understand that the root does exist. It is almost intuitive that every positive number may be thought of as having n equal factors, although one may not actually be able to get these factors. We must notice, however, that if n is odd, the answer is unique, while if n is even, there may be two sets of equal factors, since the product of an even number of negative factors is also positive.

2. The nth root of a negative number, n being a positive odd integer, is again one of its n equal factors. Remarks: The answer again is unique in that all the factors are negative, since the product of an odd number of negative factors is negative. If n is even, there is introduced the subject of complex numbers, where $i\equiv\sqrt{-1}$ is the imaginary unit. Such study, however, is not pertinent to the type of problem under discussion.

The student is supposed to have a knowledge of the laws of exponents, at least as applied to positive integers. It can be demonstrated, with such lucidity that the average student who has studied exponents with a measure of success may understand it, that these laws apply to exponents which are positive fractions whose numerators are unity and denominators positive integers. See an article by the writer in *School Science and Mathematics*, Vol. XXII, No. 3, March, 1922, paragraph VIII.

A fact which should be carefully noted

is this: when seeking the square root of a positive number, there may be certainty or uncertainty, i.e. either a unique answer or two answers. If it is known what kind of factors form the number, that is, positive or negative, then it is known definitely what kind of number the root is, whether positive or negative, since we are seeking one of its two equal factors. But if nothing is known of the kind of numbers which form the factors, then there is uncertainty as to whether the root is plus or minus. In every case, the student should be urged to discover, if possible, the nature of the factors which compose the product. If nothing is known as to the nature of the factors, then there is uncertainty, and the root may be either plus or minus. This explains why the plus and minus sign is used when finding the roots, in the algebraic sense, of a quadratic equation by formula.

In the light of these definitions and discussions, the example $[(-5)^2]^{1/2}$ has for its answer -5, which is the only answer, because we know for a certainty the kind of factors which compose the number whose square root is sought. This is confirmed by applying a law of exponents to $[(-5)^2]^{\frac{1}{2}}$. This gives $(-1)^{2\cdot\frac{1}{2}}$ or $(-5)^1$ or -5. Evidently the illustration $\sqrt{(-5)^2}$ also gives -5. The composition of the number whose square root is to be taken is definitely known, namely (-5) (-5) and its square root is one of its two equal factors, that is -5. The same holds for the second example, since a quantity raised to the one-half power is symbolized by the radical sign. But the square root of 25 is certainly plus or minus 5 since nothing is known of its history or make-up, that is, how it came to be plus 25.

If it is specified that the principal square root of a positive quantity shall be a positive number, then the number whose square root is to be taken must be positive, since the even root of a negative number is imaginary and can not be considered in this discussion.

But the application of the notion of the principal square root merely begs the question, imposes a limit on the possibility, which really is apt to create confusion in the mind of the student. The principal root must always be an even root of a positive quantity, but there may be obtained an odd root of a negative quantity. Which is its principal root? In other words, is not the notion of the principal root, in the arithmetical sense, merely a piece of impedimenta?

It would seem to be sound judgment that the only place to consider the principal square root is in the uncertainty case, and the student should not be exposed to such problems in which he can not distinguish between the certainty case and the uncertainty one. But why refer to one as the principal square root rather than the other? Both are perfectly good roots and may serve equally well. Take the problem of finding the times when a rising body, with an initial impulsion, shall be a given distance from the ground. The plus and minus signs before the radical, in the solution by formula, will give the two times when the body is a given distance from the ground, and one is not more important than the other. The real question is as to which one the greater interest attaches.

Would it not be far more valuable to ground the pupils in an intelligent understanding of fundamentals and basic, sound definitions rather than permit them to experiment too much and raise doubts? They may be acquiring information which must later be unlearned.

This article is not concerned with the roots of an equation of degree n. In the arithmetical sense, there is sought the principal real root, if the problem is considered as an equation of the form $x^n = A$ where A is the given positive number.

A Brief History of the First Twenty-five Years of the National Council of Teachers of Mathematics (Inc.)

By Edwin W. Schreiber

The National Council of Teachers of Mathematics, State Teachers College, Macomb, Illinois

It was "Knee Deep in June" in 1917 that I first met Charles M. Austin, of the Oak Park and River Forest High School, who, more than anyone else, can be called the father of the National Council of Teachers of Mathematics. We were both on the Summer Faculty of the Eastern Illinois State Normal School at Charleston. Professor E. H. Taylor (Vice-pres. 1921) was our "chief," a very considerate gentleman who has done much for the Council and the teaching of mathematics in America. Austin told me in enthusiastic terms of the Chicago Men's Mathematics Club he had helped to organize in January of 1914. He was its first President. Since I was moving from Pennsylvania to the Chicago area in the fall of 1917 he gave me a very pressing invitation to become a member of the Chicago Club. Thus in October of 1917 he introduced me to the Club as one of its new members. Here it was that I had the rare privilege of meeting and associating with many of the future leaders of the National Council, for it was in the Chicago Men's Mathematics Club that the Council had its birth.

There was John R. Clark of the Chicago Normal College, the Club President, who later became the Editor of The Mathematics Teacher, and who carried on this work for eight years, from 1921–1928. The genial professor from the University of Chicago, Herbert E. Slaught, was in the group. He later aided the Council in many capacities—as enthusiastic organizer, associate editor, chairman of the committee on incorporation, and as Honorary President. There was John A. Foberg, the tall, kindly gentleman from Crane Technical High School who served the Council as its first Secretary-Treasurer from 1920–1929. Among those present was J. O. Hassler, then of Englewood High School but now for a long time professor of mathematics at the University of Oklahoma. He has ever been a staunch friend of the Council, a tried and true servant—a Past President. There was Ernst R. Breslich from University High, a man who has served the Council for a long time. Raleigh Schorling and William D. Reeve of University High School had been members of the Club but were now working in other fields. There were many others of the Club who put their shoulders to the Council wheel but their contributions must wait for a future story of the "good old days."

Believing in the maxim—"Put first things *first*," I shall confine my story to the following FIRSTS: The FIRST meeting; the FIRST officers; the authors of the FIRST volume of The Mathematics Teacher published by the Council (Vol. XIV); the contributors to the FIRST yearbook; the service record of the officers of the Council for the FIRST twenty-five years.

The first meeting of the National Council was held on Tuesday, February 24, 1920, at the Hollenden Hotel, Cleveland, Ohio. On that day one hundred twenty-seven* enthusiastic teachers of mathematics, representing twenty states and as many local organizations, assembled at the Hollenden Hotel. The morning session began at 9 A.M. with John A. Foberg presiding. C. N. Moore, of the University of Cincinnati, spoke on "The Claims of Mathematics as a Factor in Education." Suggestions for the organization of a National Council of Teachers of Mathematics were made by representatives of each of

* If you were one of them kindly send that fact to the secretary of the National Council on a post card. The original list seems to be lost.

the following organizations: The National Council of English Teachers; The Association of Teachers of Mathematics of the Middle States and Maryland; The Central Association of Science and Mathematics Teachers, The New England Association of Mathematics Teachers. The Committee on Organization was appointed: H. O. Rugg, Chairman, E. R. Smith, C. M. Austin, Marie Gugle, J. A. Foberg. D. W. Werremeyer presided at the afternoon session. Talks were given by J. W. Young; E. R. Smith; Frank C. Touton; William Betz; Miss Marie Gugle. A constitution was presented by the Organization Committee and accepted.

The first officers of the National Council were:

President, Charles M. Austin, High School, Oak Park, Ill.

Vice-Pres., Harold O. Rugg, Lincoln School, New York, N. Y.

Secretary-Treasurer, John A. Foberg, Crane Technical High School, Chicago, Ill.

Executive Committee.

For three years

Marie Gugle, Assistant Superintendent of Schools, Columbus Ohio.

Jonathan Rorer, William Penn High School, Philadelphia, Pa.

For two years

A. Harry Wheeler, North High School, Worcester, Mass.

William A. Austin, High School, Fresno, Calif.

For one year

William D. Reeve, University High School, Minneapolis, Minn.

William E. Beck, High School, Iowa City, Iowa.

THE AUTHORS OF THE FIRST VOLUME OF THE MATHEMATICS TEACHER PUBLISHED BY THE COUNCIL (VOL. XIV)

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J. W. Young, Dartmouth College
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M. J. Newell, Evanston, Illinois, G. A. Harper, Kenilworth, Illinois
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J. C. Brown, President, State Normal School, St. Cloud, Minnesota
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W. E. Breckenridge, Head of Department, Stuyvesant High School, New York City
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J. R. Overman, State Normal College, Bowling Green, Ohio

Edward L. Thorndike, Teachers College, Columbia University
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Alfred Davis, Soldan High School, St. Louis, Missouri
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David Eugene Smith, Teachers College, Columbia University
Title: "A General Survey of the Progress of Mathematics in Our High Schools in the Last Twenty-Five Years"
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Eliakim Hastings Moore, The University of Chicago
Title: "On the Foundations of Mathematics"
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Raleigh Schorling, The University of Michigan
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William David Reeve, Teachers College, Columbia University
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William Betz, Washington Junior High School, Rochester, New York
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Frank Clapp, The University of Wisconsin
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Miss Marie Gugle, The Columbus Schools, Columbus, Ohio; and Others
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Edwin W. Schreiber, Proviso Township High School, Maywood, Illinois
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Allen, Gertrude E., University High School, Oakland, Calif., D[1]22, 23, 24.

Austin, Charles M., High School, Oak Park, Ill., P[2]20, D21, 22, 23, 24, 25, 26, V[3]-P27, 28, D30, 31, 32, 40, 41, 42.

Austin, William A., High School, Fresno, Calif., D20, 21.

Barber, Harry C., English High School, Boston, Mass., D25, 26, 27, P28, 29, D30, 31, 32, 33, 34, 35.

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*English, Harry, Franklin School, Washington, D. C., D25, 26, 27, 28, 29, 30. Died 8 June 33.

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Foberg, John A., State Department of Public Instruction, Harrisburg, Pa., S-T[5]20–28, D29.

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[1] Director.

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[5] Secretary-Treasurer.

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Schult, Veryl, Public Schools, Divisions 1–9, Washington, D. C., D43, 44, 45.
Shuster, Carl N., State Teachers College, Trenton, N. J., D44, 45, 46.
*Slaught, Herbert E., University of Chicago, Chicago, Ill., AEd28–35, HP[7]36, 37. Died 21 May 37.
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*Touton, Frank C., Univ. of Southern California, Los Angeles, Calif., D26, 27, 28. Died 1 June 36.
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*Worden, Orpha E., College of Education, Wayne University, Detroit, Mich., D21, 22, 23, 24, 25, 26. Died 9 Feb. 38.
Wren, F. Lynwood, Peabody College, Nashville, Tenn., V-P40, 41, P44, 45.

[6] Associate Editor.
[7] Honorary President.

A Merry Christmas and a Happy New Year

To All Of Our Readers!

EDITORIALS

The National Council's Silver Jubilee

It is now twenty-five years since the National Council of Teachers of Mathematics was founded at Cleveland. Since then many changes in the content and methods of teaching mathematics have been made, but there is still plenty of opportunity to improve mathematical education in the elementary and secondary schools. It is peculiarly the function of the National Council to see that such changes are made or at least to use its influence through its various meetings and publications to help to bring about reforms. However, teachers of mathematics in the field many of whom do not belong to the Council, or also being members of the Council, do not exert themselves much to improve the general situation. It is to be hoped that the next twenty-five years may bring important changes that will materially improve the content and methods of teaching mathematics in the schools.—W.D.R.

C. M. Austin's Retirement

It is appropriate that some reference be made in this silver jubilee issue of The Mathematics Teacher to the life and work of C. M. Austin, who recently retired (1944) from a long and active service as head of the department of mathematics in the Oak Park and River Forest Township High School, because Mr. Austin was one of the founders and the first president of the National Council of Teachers of Mathematics.

Mr. Austin was born near Waynesville in Warren County, Ohio, on December 13, 1874. He graduated from Waynesville High School in 1892, and began teaching in a country school on St. Valentine's Day 1893. He continued teaching and attending Ohio Wesleyan University alternately for the next ten years, graduated from Ohio Wesleyan with an A.B. degree in June 1903, and was elected to Phi Beta Kappa. He studied law in a lawyer's office in Dayton, Ohio one year, and began teaching mathematics in the high school at Milford, Ohio in September 1904, and had taught continuously since that date until he retired. He went to Oak Park and River Forest Township High School as Head of the Mathematics Department in September 1912. He helped to organize the Chicago Men's Mathematics Club in 1913, in cooperation with J. R. Clark, M. J. Newell, Raleigh Schorling, W. D. Reeve, G. A Harper, and others. He served as president of this club for the first three years of its existence. The club is still quite active and has had a wide influence on the teaching of mathematics.

Between the years 1912 and 1919, he was greatly disturbed by the increasing amount of destructive criticism hurled at mathematics. There existed no national organization to combat this criticism, or to improve the teaching of mathematics.

So, after some correspondence with leading teachers in other states, a meeting was held in Cleveland, Ohio, in February 1919.[1] There the National Council of Teachers of Mathematics was organized. Mr. Austin was chosen as the first President of the

[1] See Austin, C. M., "The National Council of Teachers of Mathematics," The Mathematics Teacher, January 1921, pp. 1–4. See also Austin, C. M., "Historical Account of Origin and Growth of the National Council of Teachers of Mathematics," The Mathematics Teacher, April, 1928, pp. 204–214. See also "Notes and News," The Mathematics Teacher, September, 1920, pp. 39–44.

new organization. At various times since, he served as Vice-President and Director. The great success and profound influence of the National Council have eminently justified the founders in their belief that such an organization was needed.

A magazine or journal of some kind was necessary for the Council to bring itself into contact with the classroom teachers; so in the summer of 1920, Mr. Austin had a conference with Professor William G. Metzger of Syracuse University who was then the Editor of THE MATHEMATICS TEACHER, which was published four times each year by an Association of Mathematics Teachers of the Eastern States and Maryland.

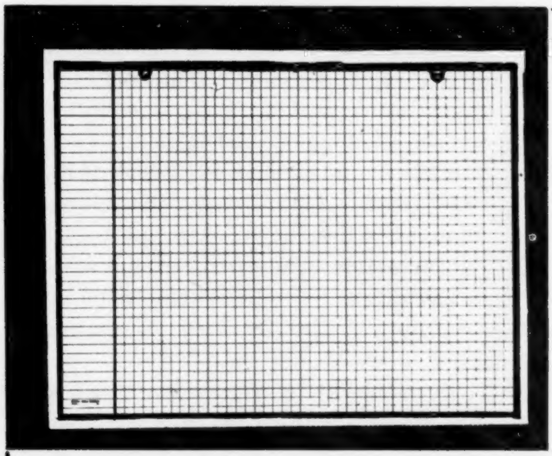
He agreed to turn over the magazine to the National Council. So, on January 1, 1921, the first issue of THE MATHEMATICS TEACHER was published by the Council. Its success has been well known. THE TEACHER has been a large factor in spreading the influence of the Council and has helped to improve the teaching of mathematics all over the country.

Believing firmly in the influence of the church, the home and the school in molding the lives of our young people, Mr. Austin has always taken an active part in Church work. At the present time he is a member of the Board of Trustees of the First Methodist Church of Oak Park.

Mr. Austin was granted an M.A. degree by the University of Chicago in 1919.

He was married to Lena Lee Morey of Battle Creek, Michigan, on December 22, 1910. The Austins have two children, Alice Jane and Churchill, now a Navy chaplain. Mr. Austin is co-author of textbooks in Plane and Solid Geometry, published by Rand, McNally & Company.

THE MATHEMATICS TEACHER wishes to extend to Mr. Austin its congratulations on his many years of useful service to the field of mathematics, and to wish him many more years of continued interest in our cause.—W.D.R.



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◆ IN OTHER PERIODICALS ◆

By Nathan Lazar
Midwood High School, Brooklyn 10, New York

American Mathematical Monthly

May 1945, vol. 52, no. 5.
1. Fite, W. Benjamin, "David Eugene Smith," pp. 237–238.
2. Rainville, E. D., "Certain Generating Functions and Associated Polynomials," pp. 239–250.
3. Mullemeister, Hermance, "Mean Lengths of Line Segments," pp. 250–252.
4. Underwood, R. S., "An Analytic Geometry for n Variables," pp. 253–262.
5. Rosenbaum, R. A., U.S.N.R., "The 'Time Unit' System in Navigation," pp. 263–264.
6. Frame, J. S., "An Approach to the Normal Curve and the Cycloid," pp. 266–269.
7. War Information: U.S.A.F.I. Courses in College Mathematics; Research Board for National Security; Educational Survey of the Armed Forces; Committee on the Teaching of the Basic Sciences; pp. 287–292.

June–July, 1945, vol. 52, no. 6.
1. Brenke, W. C., "On Generating Functions of Polynomial Systems," pp. 297–301.
2. Montgomery, Deane, "What is a Topological Group?," pp. 302–307.
3. Wall, H. S., "Polynomials Whose Zeros have Negative Real Parts," pp. 308–322.
4. Williams, G. T., "Numbers Generated by the Functions e^{e^x-1}," pp. 323–327.
5. Ford, L. R., "Some Slide Rules," pp. 328–334.
6. Thébault, Victor, "The Altitudes of a Triangle and of a Tetrahedron," pp. 335–336.
7. Hamming, R. W., "The Transcendental Character of Cos x," pp. 336–337.
8. War Information: The National Roster; American-Soviet Science Committee; Congressional Study of Higher Education; pp. 350–355.

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1. MacDuffee, C. C., "An Objective in Education," pp. 359–365.
2. Sheffer, I. M., "Convergence of Multiply-Infinite Series," pp. 365–376.
3. Frame, J. S., "Mean Deviation of the Binomial Distribution," pp. 377–379.
4. Porges, Arthur, "A Set of Eight Numbers," pp. 379–382.
5. Wedderburn, J. H. M., "On Pascal's Theorem," pp. 383–384.
6. Butchart, J. H., "Some Properties of the Limaçon and Cardioid," pp. 384–387.
7. War Information: The Crisis in Training for the Scientific Professions," 409–414.

The Duodecimal Bulletin

June, 1945, vol. 1, no. 2.
1. Beard, Ralph H., "The Do-Metric System," pp. 1–2.
2. Andrews, F. Emerson, "People with Twelve Fingers," p. 3+.
3. Camp, Kingsland, "Duodecimal Nomenclature," pp. 4–5+.
4. Andrews, F. Emerson, "Samuel Sloan."
5. Lloyd, Mary, "Dosenile Department," p. 11.
6. Editorial.

National Mathematics Magazine

April, 1945, vol. 19, no. 7.
1. Sanders, S. T., "The Evaluation of Mathematics," p. 326+.
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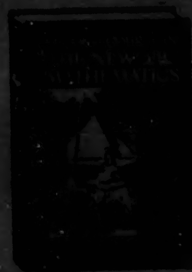
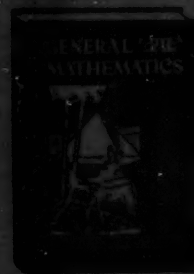

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